Quantum Signal Processing and Quantum Singular Value Transformation on U(N)

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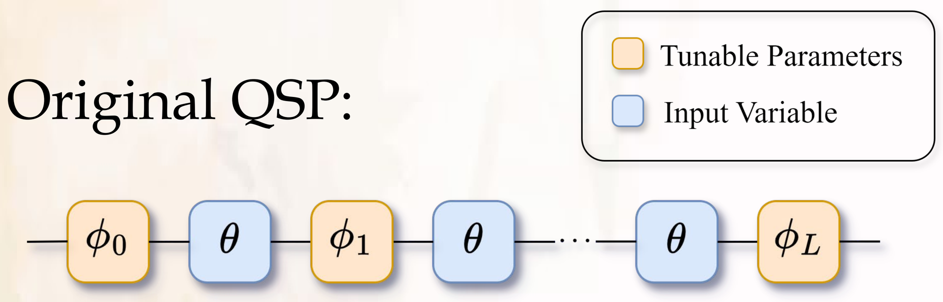
Background: U(2)-QSP

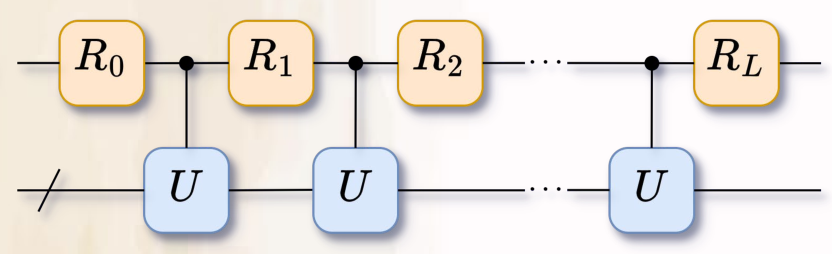
(Quantum Signal Processing)

Original QSP:



- θ: Input signal
- The blue and orange gates are rotation gates around two orthogonal axis (e.g. R_X and R_Z)

Generalized QSP:



- U: Input signal
- $R_0, \cdots, R_L$: Arbitrary single-qubit operations

In generalized QSP, there exists $R_1, \cdots, R_L \in U(2)$ such that the circuit implelements the transfrm:

$$\begin{pmatrix} f(U) & * \\ * & * \end{pmatrix}$$

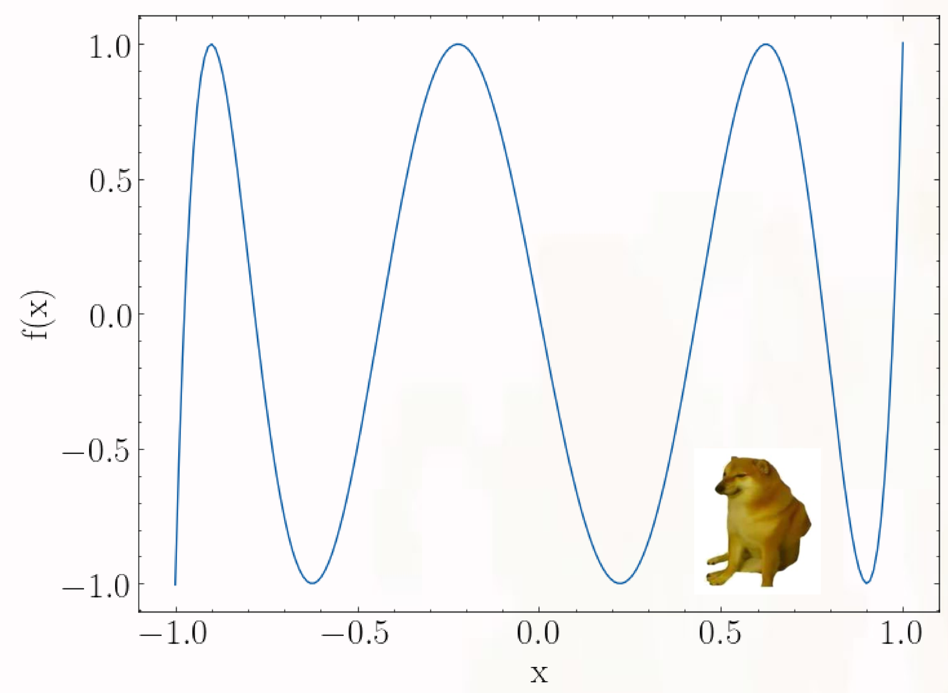
if and only if:

- f is a polynomial, $\deg(f) \leq$#Calls to U;
- $|f(z)| \leq 1$ for all $|z| = 1$.

Example applications:

- Fixed-point Search: $f(x) \sim \mathrm{sgn}(x)$;
- Linear System Solver: $f(x) \sim 1/x$;
- Hamiltonian Simulation: $f(x) \sim e^{-itx}$.

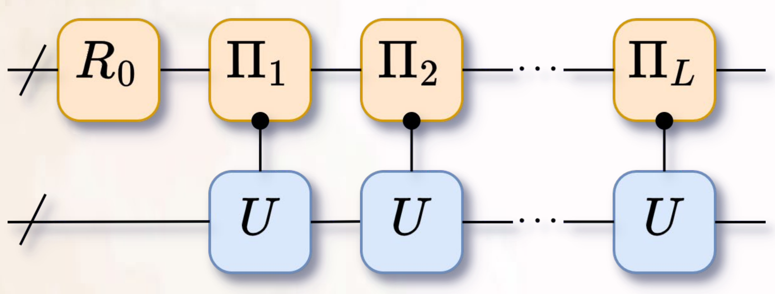
Complexity of using polynomial to approach analytic function: $d = O(\log(\frac{1}{\epsilon}))$



In U(2)-QSP/QSVT, one can only tune the circuit to realize 1 desired polynomial.
E.g.: amplitude amplification realizes the Chebyshev polynomial $\cos(\theta) \mapsto \cos((2k+1)\theta)$.

Main Result: U(N)-QSP

U(N)-QSP:



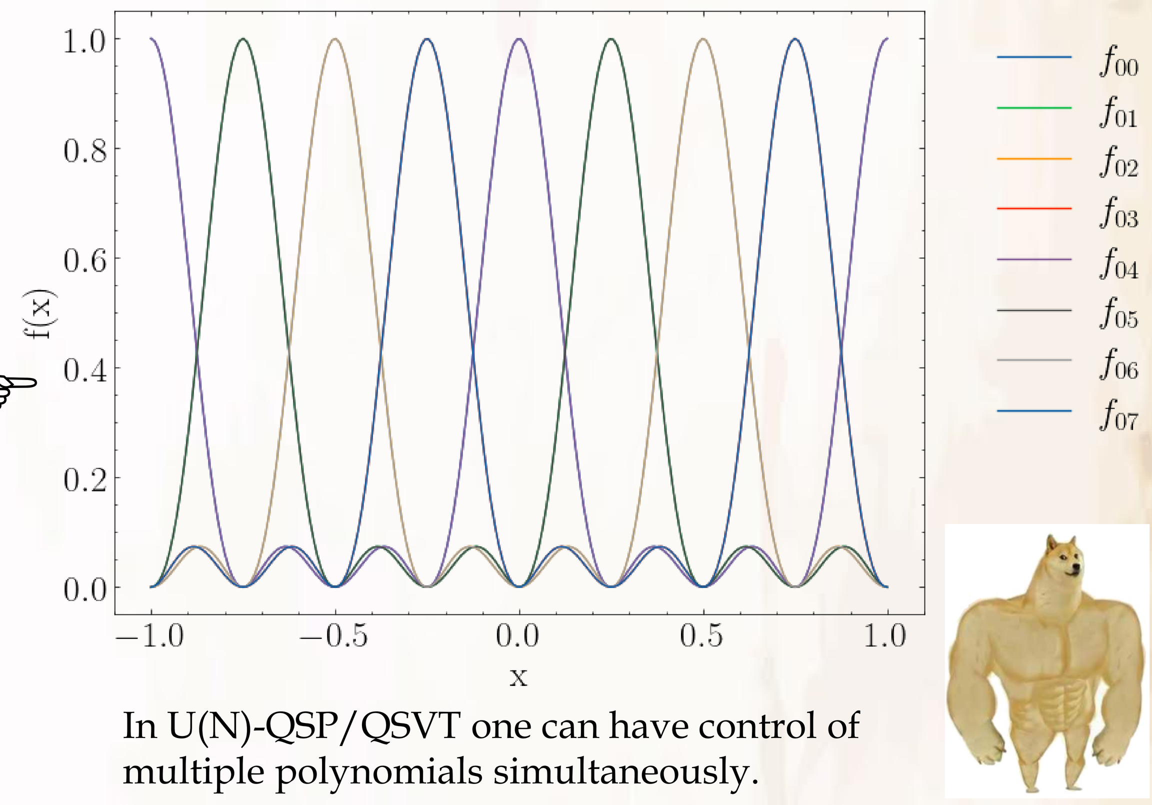
- U: Input signal
- R_0: tunable n-qubit unitray
- $\Pi_1, \cdots$: Tunable projector: $C_\Pi(U) = \Pi \otimes U + (I - \Pi) \otimes I$

In $U(N)$-QSP, there exists $R_0 \in U(N)$ and projectors $\Pi_1, \cdots, \Pi_L$ such that the circuit implelements the transfrm:

$$\begin{pmatrix} f_{00}(U) & f_{01}(U) & \cdots & * \\ f_{10}(U) & f_{11}(U) & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & * \end{pmatrix}$$

if and only if:

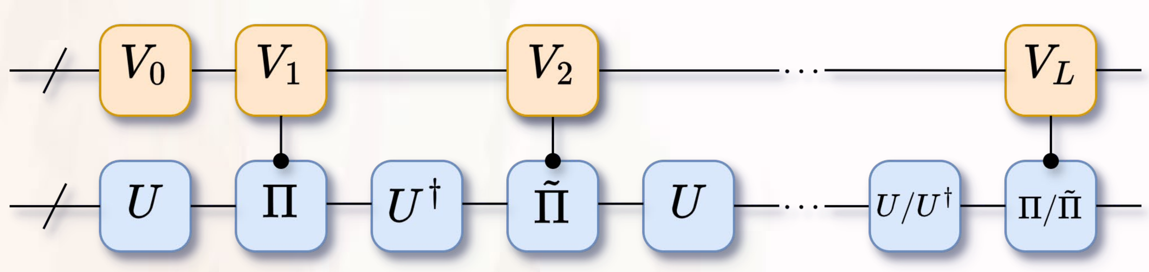
- Each f_{jk} is a polynomial, $\deg(f_{jk}) \leq$#Calls to U;
- The matrix $\{f_{jk}(z)\}$ has singular value ≤ 1 for all $|z| = 1$.



In U(N)-QSP/QSVT one can have control of multiple polynomials simultaneously.

U(N)-QSVT

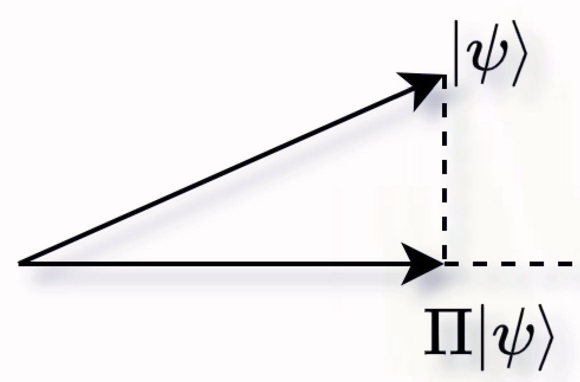
(Quantum Singular Value Transformation)



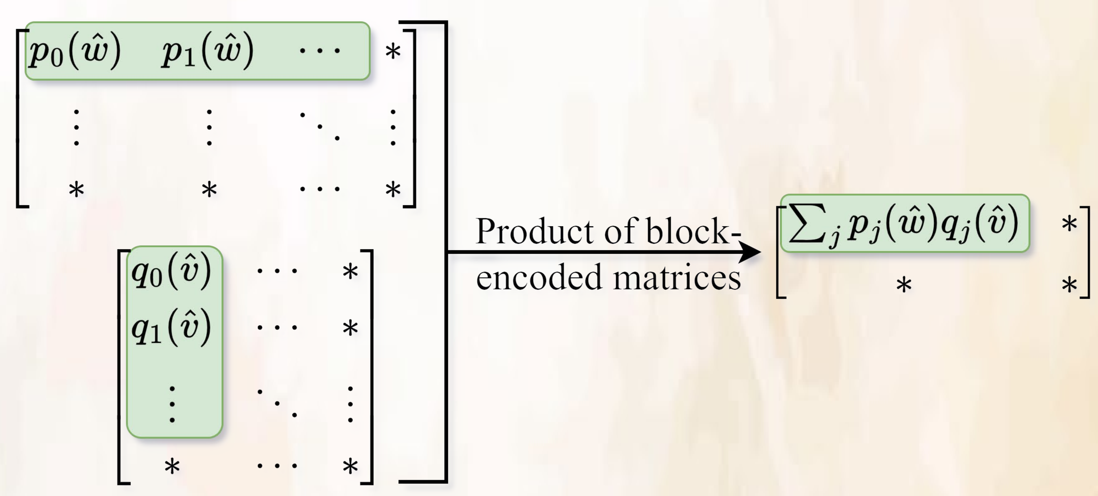
- $U = \begin{pmatrix} A & * \\ * & * \end{pmatrix}$: Block encoding of the input signal A
- $V_0, \cdots, V_L$: Tuanble U(N) operators
- Output: block encoding of multiple sigular-value polynomial transformations

$$f^{(SV)}\left(\sum_j \lambda_j |\psi_j\rangle\langle\phi_j|\right) = \begin{cases} \sum_j f(\lambda_j)|\psi_j\rangle\langle\psi_j|, & f \text{ is even} \\ \sum_j f(\lambda_j)|\psi_j\rangle\langle\phi_j|, & f \text{ is odd} \end{cases}$$

Applications



Optimal Quantum Amplitude Estimation: Estimate $x = \|\Pi|\psi\rangle\|^2$ with optimized number of state preparation oracle of $|\psi\rangle$.



Combining with other means of manipulating block-encoded matrices, like linear combination and product, U(N)-QSP could provide new approach to general multi-variate QSP[Quantum 6, 811 (2022)].

Open Questions

Can we…

- Find efficient classical algorithm to evaluate/compile $R_0, \Pi_1, \cdots$?
- Solve more sensing / metrology problems with U(N)-QSP/QSVT?
- Use U(N)-QSP/QSVT as ansatz in variational quantum algorithms / state preparation?
- Design algorithms on hybrid qubit-oscillator systems with U(N)-QSP/QSVT?
- Construct circuits for general multi-variate QSP with U(N)-QSP/QSVT being a building block?